

Non-Verbal Communication of Bilingual Students Solving Mathematics Problems

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Abstract

The view of mathematics education as an activity that involves participants in social interaction challenges what it means to know and learn mathematics, as it reveals a connection between experience and language, more specifically between words and actions. Brown (2001) argues: "spoken words are always part of some sort of action" (pp. 69-70) and raises questions such as: "How do we share our thinking with other people? How do language and symbols mediate this?" (p. 9). Habermas and Gadamer ask whether it is possible to frame all experience in language. My research analyzes the non-verbal communication of second-grade bilingual students. I argue that the richness of bilingual students' mathematical understanding requires an investigation of forms of meaning making that transcend language. The flexible use of two languages in bilingual classrooms suggests that bilingual learners may use alternative forms of communication (e.g., nonverbal communication) as equally flexibly as language. The various levels of linguistic proficiency that characterize bilingual classrooms constitute an opportunity for exploration of non-verbal communication as a way to expand what it means to communicate mathematically.

Introduction

The view of mathematics education as activity with participants in social interaction reveals a connection between experience and language, more specifically between words and actions. Brown (2001) argues: "spoken words are always part of some sort of action" (pp. 69-70). Habermas (1972) and Gadamer (1962, 1979) ask whether it is possible to frame all experience in language. According to Mason (1994), "Words generate more words in explanation, but often draw us away from the experience from which they stem" (p. 176), whereas Brousseau & Otte (1991) argue: "we obviously know more than we can tell" (p. 17).

Accordingly, my research analyzes the nonverbal communication of second-grade bilingual students solving mathematics problems. I argue that the richness of their mathematical understanding requires an investigation of forms of meaning making that transcend language. I use key ideas from

current scholarship to both explore the intersection of language, experience, and meaning, and analyze the nonverbal communication of bilingual students. Of particular concern here is the question of whether meaning can reside outside of language, on the one hand, and the relation between language and gestures, on the other.

Can Meaning Reside Outside of Language?

Not all knowledge reaches explicit linguistic form; in fact only a small portion of knowing is of this type (Davis, 1997). If knowledge manifested linguistically does not equal all knowing, how does one go about finding other forms of manifested knowledge? In bilingual mathematics classrooms, answering this question requires consideration of how students attend to mathematical ideas. These ideas, according to Brown (2001), "are not endowed with a universal meaning but rather derive their meaning through the way in which an individual attends to them" (p. 15). Data in this study suggest that students attend to mathematical ideas using both linguistic and extra-linguistic resources. Some researchers question the possibility of representing all aspects of human experience in language, while conceding that humans must encode the information from certain experiences in systems that do not use words (Huttenlocher, 1973, 1976; Alibali & Goldin-Meadow, 1993). Yet, the emphasis on verbal display of knowledge in school mathematics reflects values entrenched in the mathematics curriculum. For example, Brown (2001) and Gattegno (1988) argue that the most robust areas in the mathematics curriculum are those more easily described linguistically.

The Relation between Language and Gestures

Language and gestures are intimately linked and mediated by meaning. Gesture and speech form an integrated system in which gestures, unlike speech, are idiosyncratic, spontaneously produced, and independent from a conventional code (Alibali & Goldin-Meadow, 1993; McNeill, 1992). Kelly et al. (2002) conducted three experiments where adults were trained to interpret students' gestures as they explained solutions to problems, concluding that gestures, especially those that accompany speech, reveal information about the cognitive engagement of a student in a task, and that such information is not revealed in the student's speech. The implication is that speech and gesture together are better predictors of what students really know than either modality alone.

Similar research has found, for example, that gestures enhance messages involving communication of complex geometric shapes (Graham & Argyle,

1975), that gestures help the sender of an elaborate message get the message across (Ekman & Friesen, 1972), or that there exists a positive correlation between verbal fluency and gesture use (Cohen & Harrison, 1973).

Classifying Gestures by Function and Modality

Although gestures are idiosyncratic and difficult to classify, some researchers view them in terms of their function. Wiemann & Wiemann (1975) list six functions of nonverbal behavior: (1) repeating what has been said verbally; (2) contradicting what has been said verbally; (3) substituting a verbal message; (4) complementing what has been said verbally; (5) accenting the verbal message; and (6) regulating the flow of verbal messages.

Feyereisen & de Lannoy (1991) classify nonverbal communication according to modality (e.g., face, gaze, voice, bodily expression, use of space) and function (e.g., emotional expression, intimacy regulation, leadership, persuasion). According to these authors, "gestures and speech may constitute two ways of expressing the same idea...[and]... knowing first results from apprehending the world as 'things-of-action' and only thereafter from building 'objects of contemplation.'" (pp. 71-2). The authors hypothesize that "gestures would be triggered during speech production because meaning embodied in motor schemata would be activated" (p. 73).

Empirical Evidence of the Role of Gestures as Facilitating Verbal Comprehension

By considering the possibility that "information is not only encoded in terms of conceptual and verbal attributes or mental images, but also in terms of 'motoric representations,'" Philoppot et al. (1992) suggest that "having information encoded in more than one representation modality should strengthen the memory trace and enhance recall" (p. 195). Several studies on the nonverbal behavior of school-age children suggest the existence of reliable nonverbal cues that indicate whether students understand the content of a lesson (Allen & Feldman, 1978; Jeker, Maccoby, & Breitrose, 1964; Allen & Atkinson, 1981), or that analogous gestures, that is, gestures that agree with speech, facilitate associations in the students' memory (Bricker, 1972; Saltz & Dixon, 1982; Hulme, 1981; Hulme, Monk, & Ives, 1987; Schiaratura, 1991). Other studies have conducted experiments to investigate whether synchrony between gestures and speech facilitate recall of a spoken message (Feyereisen & de Lannoy, 1991; Woodall & Burgoon, 1981). Findings suggest that asynchrony impairs recall.

A study by Goldin-Meadow et al. (1999) reported that in classrooms

where teachers' speech was accompanied by matching gestures, students were more likely to reiterate the teacher's speech and were more able to glean problem solving strategies from that speech. Alibali & Goldin-Meadow (1993) concluded that because participants in their study produced gestures prior to receiving instruction, gestures are primal and tend to precede language behaviors that result from instruction.

With the idea that gestures can be viewed as indicators of the complex interface between language, experience, and meaning, I now turn to my study of seven second-grade bilingual students.

Methodology

Data consisted of individual interviews in a class of 7 second grade bilingual students. The school is in Austin, Texas and has a high concentration of language minority students (41.2% are Limited English Proficient, compared to state average of 14.9%) for whom a transitional bilingual program is in place. According to the teacher, who was in her first year of teaching, 2 students were proficient bilinguals and 5 were less proficient bilinguals, from the standpoint of their academic language proficiency. On three occasions, students were given two addition and subtraction problems (Appendix A). To analyze the students' nonverbal behavior, I treated data as text in order to understand its meaning. Texts, according to Bentz & Shapiro (1998), "include conversations, classroom interaction, and even clothing and fashion" (p. 107). Firstly, I categorize each gesture by functions (Wiemann & Wiemann, 1975; Feyereisen & de Lannoy, 1991). Secondly, I extended the analysis, as some gestures, particularly those occurring during verbal explanations, resisted classification by function. For these gestures, I generated conjectures that explore broader connections between gestures, experience, and meaning.

The interpretive nature of this study required consideration of validity. I addressed validity by:

a) restricting analysis to gestures that were produced during problem solving and subsequent explanation of solutions, since targeted gestures were those associated with mathematical meaning

b) viewing each tape three times. Bentz & Shapiro (1998) recommend "the returning to the object of inquiry again and again, each time with an increased understanding and a more complete interpretive account" (p. 110)

c) asking the teacher to assess the accuracy of my analysis by showing her the videotapes and my interpretation of gestures, since the teacher was more familiar with students' idiosyncratic behavior.

STUDENTS' NONVERBAL PROBLEM-SOLVING BEHAVIOR

Some students exhibited more nonverbal behaviors than others, and some non-verbal behaviors were similar across students while others differ.

Problem Solver 1 (PS1)

Day 1

In problem 1, PS1 drew 24 circles and then 24 more, indicating that the solution was 24 by moving one hand over the second set of 24 circles. This hand gesture preceded a verbal explanation.

Day 2

In problem 1, PS1 showed conceptual understanding of groups of 10 by pointing to each line of 10 circles.

Problem Solver 2 (PS2)

Day 1

PS2 wrapped her arm around her chest, looked up, and blinked repeatedly as she experienced difficulty representing the numbers in the problem. She considered direct modeling (drawing circles, then using cubes) and finally used the 100s chart (a chart with numbers 1 to 100 arranged in lines of 10 numbers).

PS2's eagerness to explain mathematical understanding verbally indicates the possibility of meaning encoded in a non-verbal system. PS2 explained that the 100s chart was easier than the cubes:

PS 2: "...because like...when they do math problems, I always find like the first number, then I find the...f...then I find the...the...I count through that one and then...then....then when I get to this one that's...that's the...like the number I said is...the...uh...the number of cookies her mom made."

Gestures helped PS2 connect a specific action (e.g., *subiendo*, *bajando*) to mathematical operations (e.g., addition, subtraction):

Interviewer: "Cuando cuentas del 45 hacia atrás, ¿qué estás haciendo en la escalera?"

PS2: "Como contando...las escaleras (escalones) [repeatedly touches her forearm with the pencil as if counting the steps]...como... "

Interviewer: "¿Hacia dónde vas en la escalera?"

PS2: "[Looks up, raising pencil] "Como de arriba, de arriba...como...a abajo."

Interviewer: "Y si cuentas del 29 al 45 ¿qué estás haciendo?"

PS2: "Subiendo [moves hand up and points up with pencil]."

Day 2

Using the 100s chart, PS2 located 46 and counted from there:

PS2: "1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21,

22...[looks up]...Ah! [Expresses frustration as she is not sure what numeral follows]."

Refocusing on the problem, PS2 imparted meaning to the operations of addition and

subtraction by using words and hand gestures.

PS2: "Oh! es el total...para atrás es quitale [points to the right with her thumb], y para enfrente es más [points to the left with four fingers together]...so...oh! ¡Ay, me confundí!...[touches her forehead and smiles]"

She resumed counting on the 100s chart, starting from 46 and counting backward:

PS2: "Pues el número que cae es la respuesta, éste cayó [pointing at 26 on the chart]."

Interviewer: "¿Qué número es?"

PS2: "Do...dos...dos...dos...son dos...dos-se...no, no lo sé [grabbed her head and looked a bit embarrassed]."

Interviewer: "¿26?"

PS2: "¡26! cayó."

Day 3

When I read the question: How many birds were there in the lake? PS2 asked:

PS2: "¿En el lago? ¿Cuántos...en total? [slid tip of pencil on the table as she said the word total]."

PS2 explained why she added:

PS2: "Sumé en el problema porque dice en total [flat hand moves horizontally from left to right as she says the word total]."

In the second problem she made the same hand movement to emphasize total:

PS2: "¿Dice cuánto duran las dos películas en total?"

This problem involved adding $55 + 49$. She tried counting from 55 on to 49. However, a 100s chart has only 100 numbers, allowing her to count up to 45 ($55 + 45 = 100$). She looked puzzled, grabbed her forehead, and finally resolved this temporary impediment by counting with her fingers until she completed 49. This moment illustrates the supportive role of gestures (finger counting) in the process of meaning-oriented activity (problem solving).

Problem Solver 3 (PS3)

Day 1

In Problem 2, PS3 checked her answer by putting out one finger for every number counted on the 100s chart, starting from 29. So 30 was 1, 31 was 2, and so on. She shook off her hands after counting 10 numbers, as if she were

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putting away groups of 10:

PS3: "Aquí van 10."

She realized that this method required too much tracking:

PS3: "¡Ay, estoy salte y salte!....[touched forehead] Or maybe if...del 45 hasta llegar al 29."

Day 2

PS3 used the calculator to subtract one set from the other.

PS3: "46...take away...30...16!"

The use of the calculator seemed to inhibit gestures.

For the second problem, she initially represented 17 with one base-ten block and 7 unit blocks. She abandoned this method and requested the calculator. I suggested persisting with this method. As she was thinking, she looked up, and bit her lower lip.

PS3: "No me sale...Estoy tratando de hacerlo con decenas y unidades en mi mente pero no puedo."

She decided to transfer the tens-and-units method to paper, and aided by finger counting, she said:

PS3: "33 [smiled],"

explaining that:

PS3: "I was counting [grabbed both sides of her forehead]...I was counting with my fingers...I started from 17 until I got to 50."

Day 3

PS3 solved problem 1 mentally and the verbal explanation was not accompanied by gestures. She solved problem 2 mentally but also used finger counting to support using the addition algorithm with relatively large numbers.

Problem Solver 4 (PS4)

Day 2

In both problems, PS4 represent the two sets with tally marks. For problem 2 he explained:

PS4: "Me faltan 33...Porque hice 50 [sweeps the entire set of tallies with his hand], y aquí hice los 17, y los encerré así para que no me confundiera...y conté estos y me salieron 33."

Problem Solver 5 (PS5)

Day 1

PS5 solved both problems mentally. For problem 2, he explained:

PS5: "Estoy contando en mi mente...de cinco en cinco para llegar más rápido."

This mental counting resulted in a wrong answer, so he sat back, moved

both hands under the desk, and looked down at his hands. Hand gestures in the form of finger counting helped him create a clearer representation than the initial mental representation. He exclaimed:

PS5: "¡16! [smiles]"

Day 2

PS5 solved problem 1 mentally. Although he moved both arms inside his shirt, perhaps because he was cold, the immediacy of his answer and subsequent explanation of it attest the mental work involved:

PS5: "16... Porque 30...46, quítale 36, al 4 le quitas 3 y se hace 1, y queda 6 y se hace 16."

With his arms still inside his shirt, PS5 solved problem 2 mentally, explaining his strategy verbally. A counting error occurred twice, and PS5 accepted the solution as correct.

Day 3:

PS5 solved problem 1 mentally. He first asked:

PS5: "¿Pero tienen que ser estos [pointing to the number of ducks] y estos [pointing to the number of geese] juntos en total?"

He solved problem 2 mentally as well, but got a wrong answer, as he converted minutes to hours. He then tried representing the problem with a clock, which he immediately gave up.

PS5: "Mejor no [smiles and pats himself on the head repeatedly]."

He returned to mental work and got the correct answer.

Problem Solver 6 (PS6)

Day 1

PS6 tried to represent the sets with interlocking cubes, but said she didn't need them because she knew how to solve the problem. As she was thinking, she put her hand between her lips, rubbed her nose and forehead, and finally counted with her fingers. Frustrated, she grabbed her forehead and said:

PS6: "I forgot!...Estoy contando...24 hasta llegar a 48."

She abandoned finger counting and decided to use the cubes again. A counting error occurred, the result of developing verbal ability to count numbers sequentially. The strategy was valid, though, as she explained:

Interviewer: "¿Cómo te salió 23?"

PS6: "Conté primero...puse 24, y luego seguí contando hasta llegar a...48."

For problem 2, PS6 said she was going to solve it the way they do in class. However, this method gave her a wrong answer, causing frustration:

PS6: "Ah! [grabbed her forehead with both hands]"

She abandoned this method and chose the cubes, which gave her the

correct solution.

Day 2

In problem 1, PS6 used the subtraction algorithm. As she performed the algorithm, she looked puzzled, scratched her forehead and eyes, but in the end she got the correct answer:

PS6: "Le quité...a 4 le quité 3 y, al 6 le quité cero, y me salió 16."

For problem 2, she used the subtraction algorithm, supported by the use of cubes.

Interviewer: "¿Por qué el primer problema lo hiciste así con números, con decenas y unidades, y el segundo lo hiciste con los cubitos?"

PS6: "[She smiles] Porque...porque éste es de quitale (problem 1)...y...y... éste es para ponerle (problem 2)."

Interviewer: ¿Para ponerle? ¿Y qué pusiste?

PS6: "Huh...huh [looks back at the problem]...cin-cuen-ta...oh, es quitale...no [shook her head]...es súmale...50. A 17, quit-...súmale 33 [shook her head again]."

Day 3

In both problems, she used the addition algorithm, supported by finger counting. In problem 2, she expressed surprise by the size of the number that she got as the answer:

PS6: "¡Me salió mucho! [Covered her mouth with her hand]."

Problem Solver 7 (PS7)

Day 1

In problem 1, PS7 represented the sets with tally marks.

PS7: "Le puse...24, y...des-... de aquí le seguí hasta 40...digo 48, y...¿Le, le debo poner el total? [Moved hand across the rows of tally marks repeatedly]"

For problem 2, he explained:

PS7: "De aquí, hasta 29, 29 y luego...[Counted the 29 marks]... hasta aquí, y luego le comencé a contar aquí [Rotated his position in the chair to count the remaining marks], conté estos y eran 16 para subir hasta...hasta...[Tapped with pencil on the last tally mark]."

Day 2

In both problems, PS7 represented the sets with circles. In problem 1, he realized he misread the numbers:

PS7: "Primero hice círculos...[looked back at the problem]... ¿46? ¡Yo puse 49! [grabbed his head and smiled, redrew the sets and got the correct answer, 16]"

In problem 2:

PS7: "Primero puse los 17, y conté uh [Swept the whole set with his

hand]...los 50...y puse esta rayita para saber...donde (unintelligible)."

Day 3

In problem 2, PS7 represented the sets with tally marks. As he moved into larger numbers, he experienced difficulty naming the numbers sequentially. He solved this inconvenience in the following manner:

PS7: "I need a...[Made the shape of a square with his hands]...the 100s chart."

He used the 100s chart not for counting, but as a reference to know the next number name in the sequence.

Results and Discussion

A remarkable result was that the students-with very few exceptions-solved all problems correctly. To do this, they imparted meaning to each task by mutually supporting their verbal and nonverbal behaviors. For example, when students realized a counting error, or a strategy became inefficient, or during strategy selection, five students (PS2, PS3, PS5, PS6, and PS7) grabbed, patted, touched, or rubbed their head. This behavior exemplifies the emotional expression function of gestures (Feyereisen & de Lannoy, 1991). Six students (PS1, PS2, PS3, PS4, PS6, and PS7) produced the verbal behavior of counting simultaneously with the nonverbal behavior of pointing or touching each element counted. This behavior illustrates the complement function of gestures and supports the hypothesis that gestures and speech are correlated in meaning (Alibali & Goldin-Meadow, 1993; McNeill, 1992; Graham & Argyle, 1975; Ekman & Friesen, 1972; Cohen & Harrison, 1973). Four students (PS1, PS2, PS4, and PS7) showed a sweeping hand movement to indicate the total in a problem, a behavior that repeats or adds emphasis to verbal behavior (Weimann & Weimann, 1975). Two students (PS2 and PS7) pointed to numbers for which they did not recall the name, and one student (PS7) made the shape of a square with both hands to request the 100s chart. Both behaviors illustrate the substitution function of gestures (Weimann & Weimann, 1975). One student (PS6) shook her head no as she was deliberating whether to add or subtract in a problem, which illustrates the accenting function of gestures (Weimann & Weimann, 1975).

The distinctiveness of some gestures required extending this functional analysis. For example, PS1's indication of a total with a hand gesture before articulating any verbal explanation exemplifies the primacy of gestures over verbal behavior, a hypothesis that Feyereisen & de Lannoy (1991) frame as "apprehending the world as things-of-action" (p. 71). In Day 1, PS2's eagerness to communicate meaning verbally suggests the possibility of

information encoded in a nonverbal system (Alibali & Goldin-meadow, 1993), or a system that encodes intuition-related information (Gattegno, 1988). While some students (PS2) used gestures to impart meaning in the form of a familiar action (e.g., para arriba, para abajo, para atrás es quitale y para enfrente es más) to otherwise abstract mathematical operations (e.g., addition and subtraction), others exhibited less of these gesture-action associations, for example when they solved the problems mentally (PS5) or when they used a method that inhibited gestures, such as a calculator (PS3). An example of gestures regulating cognitive activity is shown by PS3 who shook off her hand after she counted 10 numbers, symbolically indicating the completion of a mathematical task: counting by tens.

Attention to gestures during problem-solving activity in bilingual classrooms offers multiple benefits. If gestures are closer to intuitive knowledge than to academic knowledge, an implication of this conjecture is that attention to gestures during instruction can foster intuitive knowledge, which in turn may impart meaning to academic knowledge. Perhaps the greatest benefit is a better-informed assessment of bilingual students' mathematical understanding and ability than that resulting from a focus on linguistic ability. Given the unpredictable distribution of this population in classrooms, and their invisibility as learners with meaningful thinking, such assessment is much needed.

To communicate their thinking to others, the problem solvers in this study mediated meaning through language and gestures. Such a result substantiates the argument of whether meaning can reside outside of language. The fact that all participants in my study (teacher, students, and researcher) were of the same linguistic (English-Spanish bilingual) and ethnic (Mexican or Mexican-American) background suggests that gestures were uncontaminated by unequal power relations. However, it is imperative to consider (and conduct research in) contexts where unequal power relations affect the ways bilingual students communicate their thinking to others. All students, bilingual or not, have relationships to knowledge. Gestures, I argue, may exemplify what Bourdieu (1977) describes as "the intangible nuances of manner and style which are the imperceptible and yet never unperceived manifestations of the individual's relationship to knowledge" (p. 338). In classrooms, teachers perceive students' relationship to knowledge, and react to it in ways that either validate and extend or invalidate and restrict such student-knowledge relationships. Meaning-oriented analysis such as the one proposed herein is impossible if we subscribe to a purely linguistic analysis. All that accompanies language (e.g., pauses, intonation, rectification,

silences, false starts, facial and hand gestures, etc.) contribute to reveal the constitution of meaning.

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Appendix A

Subtractive Situations

Type of Problem: Comparison

Given on Day 1

1. Jake made 24 cookies. His mom also made cookies. There were 48 cookies altogether. How many cookies did Jake's mom make?

2. You and your best friend are inside a castle. The castle has a high tower with a stair that goes all the way to the top. You and your friend decide to go to the top of the tower, but you get there first. You counted the steps and you know there are 45 steps. When you reach the top, you call your friend on the cell phone and he says: I am on step number 29. How many more steps do I need to go? What should you answer to your friend?

Subtractive Situations, Separate (1) and Compare (2)

Type of Problem: Action Cue

Given on Day 2

1. Kira had 46 pansies in her garden. She picked up 30 to give to her father. How many did she have left?

2. You are collecting fireflies in a glass jar to see them turn on at night. You want to have 50 so you can fill a big jar. You have caught only 17. How many more do you need?

Additive Situations

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Type of Problem: Static Set Relation

Given on Day 3

1. One day there were 26 ducks and 36 geese in the lake. How many birds were there in the lake?
2. Today you will see two Pokemon movies with your friends. One movie lasts 55 minutes and the other movie lasts 49 minutes. How many minutes do the two movies last altogether?